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Scattered results in 2D string theory

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Abstract: The nonperturbative $1 \to N$ tachyon scattering amplitude in 2D type 0A string theory is computed. The probability that N particles are produced is a monotonically decreasing function of N whenever N is large enough that statistical methods apply. The results are compared with expectations from black hole thermodynamics.

Keywords: D-branes, Black Holes, Matrix Models.

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Contents

1. Introduction

String theory outside the critical dimension has proven a fruitful subject of investigation, yielding many valuable insights. The introduction of such "non-critical" strings by Polyakov [1] highlighted the importance of local scale invariance as a consistency condition on the worldsheet QFT describing string propagation. Indeed, noncritical string models are now understood as a particular class of backgrounds of ordinary, "critical" string theory with asymptotically linear dilaton:

$$G_{\mu\nu} \sim \eta_{\mu\nu} \,, \qquad \Phi \sim n \cdot X \tag{1.1}$$

where $n^2 = \frac{26-D}{6\alpha'}$ for the bosonic string, and $n^2 = \frac{10-D}{4\alpha'}$ for the fermionic string. Perturbative string theory in such a background is self-consistent if the dynamics avoids the region of strong coupling. One way to accomplish this (for $D \leq 2$) is to turn on a closed string tachyon background

$$T = \mu \, e^{\alpha n \cdot X} \,. \tag{1.2}$$

The resulting worldsheet potential pushes strings away from strong coupling.

The formulation of non-critical string models in target dimension $D \leq 2$ in terms of integrals over random matrices [2, 3, 4] provided the first insights into stringy non-perturbative effects [5, 6]. Indeed, recent work [7]–[12] has established that the matrix model describes an open string dual of non-critical string theory background (1.1)–(1.2), in which the matrix eigenvalues are D-branes of the non-critical string.

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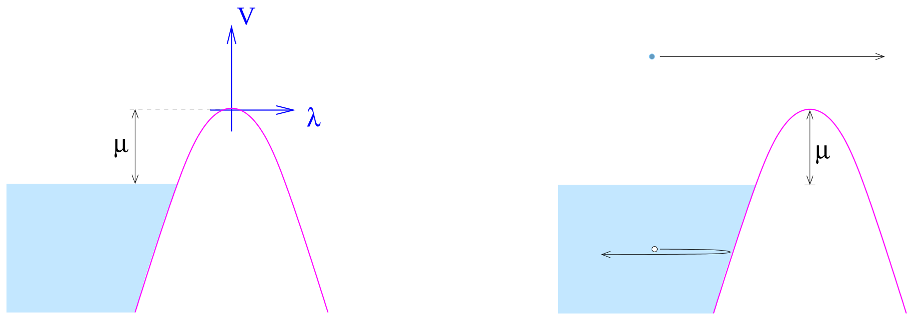

Figure 1: The matrix model for the bosonic string: (*a*) The metastable "ground state"; (*b*) The puzzle introduced by high energy scattering.

Thus these string backgrounds provide particularly tractable examples of the correspondence between gauge theory and gravity uncovered in recent years [13] (for a review, see [14]). Here, both sides of the correspondence are amenable to precise calculation. Namely, on the matrix model/open string/gauge theory side, the matrix path integral is exactly solvable. On the closed string/gravity side, the correlation functions of the worldsheet CFT of the non-critical string can be computed exactly using conformal bootstrap methods [15]–[19] (for a review, see [20]). The agreement between the two approaches of all quantities thus far computed provides strong evidence that the two formulations are equivalent.

On the closed string side, another interesting non-critical string background is the 2D black hole [21, 22]

$$\begin{aligned} ds^2 &= \frac{k}{\alpha'}[dr^2 - \tanh^2 r\, dt^2] \\ \Phi &= \Phi_0 - 2\log[\cosh r] \end{aligned} \tag{1.3}$$

with $k = 9/4$ for the bosonic string, and $k = 5/2$ for the fermionic string. This geometry is in the class of asymptotically linear dilaton backgrounds (1.1), and the euclidean geometry admits an exact worldsheet CFT description as the gauged WZW model $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$. These results raise the possibility of a non-perturbative and perhaps solvable matrix model formulation of 2D black hole dynamics.

An obstacle to the realization of this idea in the bosonic string is the non-perturbative ambiguity in the definition of the corresponding matrix model. In the latter, the matrix eigenvalues are free fermions in an inverted quadratic potential, with only the metastable states on one side of the barrier filled (see figure 1*a*). The interpretation of eigenvalues in the "ground state" tunneling to the other side of the barrier, or simply vaulting over the barrier in a high energy process as in figure 1*b*, remained unclear. Since black hole formation is expected to arise through such high energy processes (c.f. [23] for a discussion in the present context), the status of black holes in the matrix model also remained unclear.

Now it is understood that the stable ground state, where eigenvalue fermions fill both sides of the harmonic barrier, is a matrix model formulation of the superstring — specifically, its type 0B incarnation [11, 12] (see figure 2*a*). It seems appropriate to revisit the question of black hole formation in the matrix model.

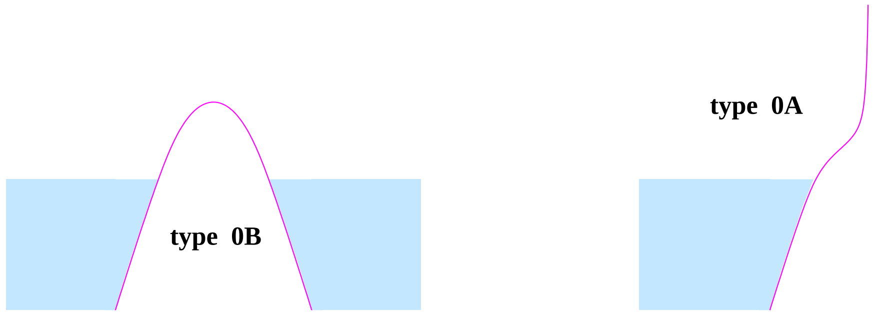

Figure 2: The matrix model eigenvalue potentials and ground state configurations for the (a) type 0B, and (b) type 0A fermionic string.

For reasons of technical simplification, we will consider the related type 0A matrix model, which allows us to sidestep various subtleties associated to the intercommunication of the two sides of the harmonic barrier in the 0B model [24]. The eigenvalue potential of the 0A model [12, 25]

$$V = -\frac{\lambda^2}{4\alpha'} + \frac{q^2 - 1/4}{2\lambda^2}\,, \tag{1.4}$$

depicted in figure 2b, has eigenvalues restricted to the half-line $\lambda < 0$. The parameter q is the RR charge of the background. For $q = 0$ the black hole solution of (1.3) applies (with $k = 5/2$), and can again be described on the worldsheet as the $\mathcal{N} = 1$ supersymmetric $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ coset model. The more general black holes with $q \neq 0$ have been studied in [26]–[29].

Can one form black holes in 2D string theory? In any situation where black holes form, one expects them to constitute the generic intermediate states due to their large density of states, and therefore to dominate the behavior of the S-matrix. Thus we would like to ask whether the S-matrix of 2D string theory has any characteristics that one would associate with the appearance of black holes as intermediate states. In higher dimensions, this is indeed the case in semiclassical gravity; the generic result of a very energetic collision is the formation of a black hole.

Nevertheless, two arguments suggest that black holes do not form in 2D string theory.

1. First, the matrix model has infinitely many conserved quantities, since the underlying dynamical objects are free fermions. Any black hole will correspond to rather specific values of these conserved quantities, and therefore the generic initial state having other values for these quantities will have small overlap with the black hole.

2. In other examples of the gauge/gravity correspondence, the formation of black holes is associated with a deconfinement transition in the gauge theory, in which one accesses non-singlet degrees of freedom in the gauge theory [30]. These degrees of freedom are projected out of the matrix model for 2D string theory, suggesting that there is no regime in which the dominant configurations in the dynamics can be associated to black holes.

These two issues are related. The appearance of free fermions and their associated hierarchy of conserved quantities is directly related to the absence of non-singlet degrees of freedom in the matrix model, which would be needed to provide the exponentially large degeneracy of black hole states. The projection onto singlets is consistent with the Gauss law of the gauge theory on D0-branes, which provide the worldsheet description of matrix eigenvalues pulled out of the Fermi sea [12].[1]

Even so, it would be nice to verify this conventional wisdom with a concrete calculation, to see if there are any indications of an object with the characteristics of a black hole which participates as an intermediate state in the dynamics. For example, it might be possible for a forming black hole to shed via tachyon radiation the collection of conserved charges along with some fraction of the infall energy, leaving behind a long-lived black hole intermediate state.

Below we will examine high energy scattering processes in the matrix model, using the formalism of [32] for the exact S-matrix. Of course, the high energy behavior of the S-matrix is of intrinsic interest, independent of the question of whether black holes are involved in the scattering. We wish to look for the probability to find N particles in the final state, to see if there is a correspondence with the expected decay spectrum of a 2D black hole. We will examine in detail the $1 \to N$ scattering amplitude, but we expect other amplitudes involving a finite number of high energy incoming particles to behave similarly. Is there any feature in the outgoing spectrum that we might identify with a black hole intermediate state? We will show that, for N more than a few, the probability to produce N particles is a rapidly decreasing function of N, with no feature in the range $N \sim \omega$ that would be characteristic of black hole intermediate states.

2. The black hole in type 0A gravity

Before embarking on an analysis of scattering in the matrix model, let us consider what result would be predicted by the appearance of 2D black holes as the generic intermediate state.

At lowest order in α', the effective action of type 0A string theory admits a class of RR charged black hole solutions [26]–[29]. In Schwarzchild-like coordinates, the geometry can be written (for $\mu = 0$, i.e. no tachyon field background)

$$\begin{aligned} ds^2 &= -l(\phi)dt^2 + \frac{d\phi^2}{l(\phi)} \\ \Phi &= \kappa\phi \end{aligned} \tag{2.1}$$

where $\kappa = \sqrt{2/\alpha'}$, and $l(\phi)$ is given by

$$l(\phi) = 1 + e^{2\kappa(\phi - \phi_{\rm H})} \left[2\kappa e^{-\varepsilon}(\phi - \phi_{\rm H}) - 1\right] . \tag{2.2}$$

[1] Even if non-singlet states were somehow allowed, their energy cost becomes infinite in the continuum limit of the matrix model [31].

Here, $\phi = \phi_{\rm H}$ is the location of the black hole horizon; the horizon of the extremal black hole is located at

$$\phi_e = \frac{1}{2\kappa} \log \left[\frac{32\pi}{q^2} \right], \tag{2.3}$$

and

$$\varepsilon = 2\kappa(\phi_e - \phi_{\rm H}) \tag{2.4}$$

is a measure of the departure from extremality.

In terms of these quantities, the Hawking temperature and ADM mass above extremality are given by

$$\begin{aligned} T &= \frac{\kappa}{2\pi}(1 - e^{-\varepsilon}) \\ E_{\rm ADM} &= 2\kappa e^{-2\kappa\phi_e} \left[e^{\varepsilon} - 1 - \varepsilon \right]. \end{aligned} \tag{2.5}$$

Note that the temperature rises smoothly from zero at extremality to a limiting temperature of order one in string units. The high mass behavior is thus Hagedorn thermodynamics, with an entropy

$$S_{\rm BH} \sim \beta_{\rm H} M \tag{2.6}$$

where $\beta_{\rm H} = 2\pi\sqrt{\alpha'/2}$.

The Hawking temperature is related to the actual spectrum of radiation seen at infinity via so-called greybody factors (c.f. [33]). The potential seen by a Hawking quantum as it travels away from the horizon acts as a filter and distorts the spectrum. Let us estimate the magnitude of such effects.

The equation of motion for the closed string tachyon $\widetilde{T} = e^{-\Phi}T$ in this background is

$$\left(l \frac{d}{d\phi} \right)^2 \widetilde{T} + (\omega^2 - V)\widetilde{T} = 0\,, \tag{2.7}$$

where

$$\begin{aligned} V &= 7\kappa l l' - 15\kappa^2 l(l-1) \\ &= \kappa^2 l e^{2\kappa(\phi - \phi_{\rm H})} \left[1 + 14 e^{-\varepsilon} - 2\kappa e^{-\varepsilon}(\phi - \phi_{\rm H}) \right]. \end{aligned} \tag{2.8}$$

A sketch of this effective potential is shown in figure 3.

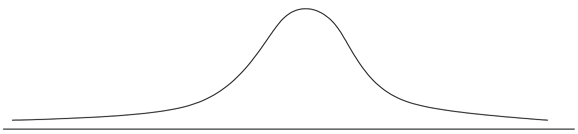

Figure 3: The effective potential seen by a tachyon perturbation in the 0A black hole background, as a function of the "tortoise coordinate" $x = \int l^{-1} d\phi$.

The height of the effective potential is of order one in string units. The transmission amplitude rises to unity for incident energies far above the barrier height, and vanishes smoothly as the incident energy goes to zero. At any rate, for black holes of mass above extremality $E_{\rm ADM} \gg \kappa$, where the Hawking temperature is of order $T \sim \kappa/2\pi$, the greybody effects amount to a factor of order one. It is only for near-extremal black holes, where the typical Hawking quantum has an energy small compared to the barrier height, that greybody effects will be significant.

One can also consider black hole formation in the context of the low-energy effective field theory. The gravitational back-reaction of an infalling tachyon pulse propagating according to (2.7) naively results in black hole formation for any sufficiently energetic pulse.[2] The horizon moves out to large ϕ, and therefore weak coupling, with increasing energy. The bulk of the pulse is not backscattered by the effective potential V, equation (2.8). Provided that α' corrections to the low-energy field equations do not change the qualitative structure deduced from the leading order equations, the black hole is the generic intermediate state for any highly energetic initial state — once tachyons fall inside a trapped surface, the black hole will form. The trapped surface forms at weak coupling for sufficiently large energy, and so the classical field equations are reliable.

The qualitative picture that then emerges is as follows: Energetic incoming states form black holes. For black hole masses far above extremality, $E_{\rm ADM} \gg \kappa q^2$, the Hawking temperature $T_{\rm H}$ is of order one in string units. The decay of the black hole will proceed smoothly via the emission of quanta whose energy is of order one in string units, until $E_{\rm ADM} \sim \kappa q^2$. At this point the character of the decay changes; the temperature varies with the mass, smoothly decreasing with as the black hole evaporates toward extremality. We can make these endpoint effects parametrically small by considering black holes whose initial mass is large compared with the charge. The expectation then is that the typical number of quanta emitted in a black hole decay will be of order the mass in string units,

$$\langle N \rangle \sim \frac{E_{\rm ADM}}{T_{\rm H}} = \frac{E_{\rm ADM}}{\kappa/2\pi}\,. \tag{2.9}$$

It should however be emphasized that the geometry (2.1)–(2.2) is strongly curved in string units, and the above picture of its properties could be strongly corrected by stringy effects. It is somewhat reassuring in this regard, that for $q = 0$ the black hole admits an exact CFT description as an $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ gauged WZW model. Even so, this may also be misleading, as we will discuss in the final section.

3. The type 0A S-matrix

The asymptotic states of type 0A string theory are perturbations of the tachyon field, realized in the matrix description as disturbances of the eigenvalue density. To evaluate the S-matrix, one is instructed to perform a kind of LSZ reduction on eigenvalue density correlators, taking the point of evaluation of the corresponding fermion bilinears to $|\lambda| \to \infty$, $t \to \pm\infty$ and extracting the coefficient of the asymptotic behavior. The connected correlation function of the asymptotic eigenvalue density perturbations consists of ring diagrams of the corresponding fermion bilinears; an example of $1 \to N$ scattering is depicted in figure 4.

[2] As in the study of dilaton gravity coupled to conformal matter [34], we would have to consider a incoming tachyon pulse localized along $\mathcal{I}^-$ in order to have a finite energy density which is sufficiently localized to create a black hole. This can be achieved by making a tachyon wave packet instead of the plane wave state which we will consider in the following sections. However, this does not change the qualitative feature of the number distribution in the final state discussed in section 5 (see also footnote 4). We would like to thank A. Strominger for raising this issue.

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A recipe for the result of this calculation was given in [32] (and reviewed in [3]). One picks from the products of fermion propagators the terms in which the wildly oscillating phases $\exp[\frac{i}{2}\lambda^2]$ cancel; one then extracts the coefficients of the asymptotic behavior of the density field

$$\rho(\lambda, t) \sim \frac{1}{\lambda}\exp[-i\omega(t \pm \log|\lambda|)] \tag{3.1}$$

to obtain the scattering matrix element. For each fermion reflecting off the potential wall near the origin in λ space, there is a factor of the reflection coefficient $R(\omega)$ for particles and $R^*(\omega)$ for holes. The reflection coefficient of type 0A is given by[3]

$$R(\omega) = \left|\frac{q^2}{4} - \frac{1}{16} + \mu^2\right|^{-i\omega} \frac{\Gamma(\frac{1}{2} + \frac{q}{2} + i\omega - i\mu)}{\Gamma(\frac{1}{2} + \frac{q}{2} - i\omega + i\mu)}\,. \tag{3.2}$$

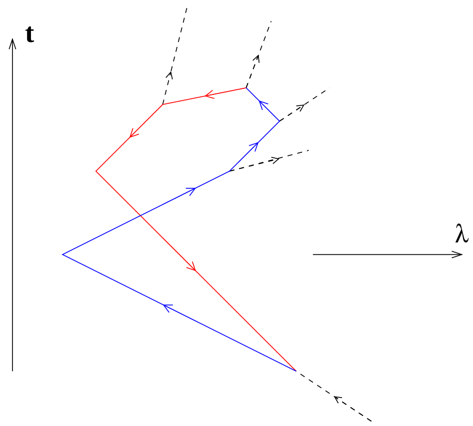

Figure 4: $1 \to N$ scattering in the matrix model. The incoming tachyon (dashed line) splits into a particle (blue) and hole (red), which reflect off the potential wall. The particle-hole pair decays back toward the Fermi surface via the emission of final state tachyons.

In comparing to worldsheet results, there are in addition certain “leg-pole factors” relating the matrix model density perturbation to the string tachyon [35]. These are energy-dependent phases for on-shell scattering processes, which therefore affect quantities such as the time delay of an outgoing particle. However, they will not affect the outgoing particle distribution as a function of energy, which is the object we study here. Therefore we henceforth ignore these factors.[4]

Our operating hypothesis, as explained in the last section, is that any sufficiently energetic incoming state causes the formation of a black hole. We will thus concentrate on the $1 \to N$ amplitude for simplicity (a partial analysis of the $2 \to N$ amplitude is contained in the appendix). The momenta of the external states then divide into two groups: The set S that attach to the propagating fermion line, and therefore have positive frequency; and the set $\overline{S}$ that attach to the propagating hole line,[5] and therefore have negative frequency. According to the prescription outlined above, the scattering amplitude is

$$\mathcal{A}_N(\omega|\omega_i) = \sum_{S\subset\{1,\ldots,N\}} (-1)^{|S|} \int_{\omega_S}^{\omega} d\xi\, R(\omega-\xi)R^*(-\xi) \tag{3.3}$$

where $\omega_S = \sum_{l=1}^{k} \omega_{j_l}$ and $|S| = k$ for $S = \{j_1, \ldots, j_k\}$.

[3] We set $\alpha' = 2$.

[4] The leg-pole factors could potentially alter the result when considering a wave packet of incoming tachyon. The width $\delta\omega$ of the wavepacket determines the localization in position space. One can choose the width much smaller than the string mass, so that the leg pole factor is constant over the width of the wavepacket in frequency space, and yet much larger than the inverse of the time scale T that the entire wavepacket spends in the strong coupling region, behind the point in ϕ where the low energy field equations would predict that a horizon forms. The variation of the amplitude is smooth over the width of such a wavepacket, and so the result of scattering such a localized wavepacket will not differ substantially from that of a pure plane wave.

[5] There is always one tachyon emitted at the point where the particle and hole lines meet in the final state; in our conventions, this tachyon belongs to the set $\overline{S}$.

Plugging the explicit form of reflection coefficient (3.2) into (3.3), $\mathcal{A}_N$ is rewritten as

$$\mathcal{A}_N = \left|\frac{q^2}{4} - \frac{1}{16} + \mu^2\right|^{-i\omega} \sum_S (-1)^{|S|} \int_{\omega_S-\mu}^{\omega-\mu} d\xi F(\omega-\xi)F(\xi) \tag{3.4}$$

where we defined $F(\xi)$ by

$$F(\xi) = \frac{\Gamma(\frac{1}{2}+\frac{q}{2}+i\xi)}{\Gamma(\frac{1}{2}+\frac{q}{2}-i\xi)}. \tag{3.5}$$

Dropping the overall phase factor $|\frac{q^2}{4} - \frac{1}{16} + \mu^2|^{-i\omega}$ and changing the integration variable as $\xi \to \frac{\xi+\omega}{2}$, the amplitude $\mathcal{A}_N$ in (3.4) becomes

$$\mathcal{A}_N = \frac{1}{2}\sum_S (-1)^{|S|} \int_{\omega_S-\omega_{\bar{S}}-2\mu}^{\omega-2\mu} d\xi F\left(\frac{\omega-\xi}{2}\right) F\left(\frac{\omega+\xi}{2}\right). \tag{3.6}$$

In [32], a saddle point approximation was applied directly to this expression (3.6) of amplitude. However, it is more useful to perform a summation over S before doing the saddle point approximation. For this purpose, we utilize the trick of inserting "1" into the integral:

$$1 = \int_{-\infty}^{\infty} d\nu\delta(\nu-\xi) = \int_{-\infty}^{\infty} \frac{d\nu dt}{2\pi} e^{it(\nu-\xi)}. \tag{3.7}$$

Then (3.5) becomes

$$\begin{aligned} \int_{\omega_S-\omega_{\bar{S}}-2\mu}^{\omega-2\mu} d\xi f(\xi) &= \int_{\omega_S-\omega_{\bar{S}}-2\mu}^{\omega-2\mu} d\xi \int_{-\infty}^{\infty} \frac{d\nu dt}{2\pi} e^{it(\nu-\xi)} f(\nu) \\ &= \int_{-\infty}^{\infty} \frac{d\nu dt}{2\pi} e^{it(\nu+2\mu)} f(\nu) \frac{1}{-it}\left(e^{-it\omega} - e^{it(\omega_{\bar{S}}-\omega_S)}\right) \end{aligned} \tag{3.8}$$

where we defined $f(\xi) = F(\frac{\omega-\xi}{2})F(\frac{\omega+\xi}{2})$. Now we can perform the summation over S by using the relations $\sum_S (-1)^{|S|} = 0$ and

$$\sum_S (-1)^{|S|} e^{it(\omega_{\bar{S}}-\omega_S)} = \prod_{j=1}^{N} (e^{it\omega_j} - e^{-it\omega_j}). \tag{3.9}$$

Finally, we arrive at a compact form of $\mathcal{A}_N$

$$\mathcal{A}_N = i^{N-1} \int_{-\infty}^{\infty} \frac{dt}{2t} G(t) e^{2it\mu} \prod_{j=1}^{N} 2\sin\omega_j t \tag{3.10}$$

where $G(t)$ is given by

$$G(t) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} e^{it\nu} F\left(\frac{\omega-\nu}{2}\right) F\left(\frac{\omega+\nu}{2}\right). \tag{3.11}$$

Note that $G(t)$ is an even-function of t. The parameter t can be interpreted physically as the time difference between the bounce of the particle and the bounce of the hole.

In the following, we will set $\mu = 0$ for simplicity (its effects are easily restored). Our expression (3.10) reproduces the well-known selection rule that when $\mu = 0$ the amplitude vanishes for $N =$ even [36]. Below we will study the high energy behavior of the non-vanishing $N =$ odd amplitude:

$$\mathcal{A}_N = i^{N-1} \int_0^\infty \frac{dt}{t} G(t) \prod_{j=1}^{N} 2 \sin \omega_j t \,. \tag{3.12}$$

3.1 Zeroth order approximation

Suppose $G(t)$ is independent of t: $G(t) = G(0)\ \forall t$. Then the amplitude (3.12) becomes

$$\begin{aligned} \mathcal{A}_N &= G(0) i^{-1} \int_0^\infty \frac{dt}{t} \prod_{j=1}^{N} (e^{it\omega_j} - e^{-it\omega_j}) \\ &= G(0) \sum_S (-1)^{|S|} \int_0^\infty \frac{dt}{t} \sin(\omega_{\bar{S}} - \omega_S) t \\ &= \frac{\pi}{2} G(0) \sum_S (-1)^{|S|} \epsilon(\omega_{\bar{S}} - \omega_S) \\ &= \pi G(0) \sum_S (-1)^{|S|} \theta(\omega - 2\omega_S) \,, \end{aligned} \tag{3.13}$$

where $\epsilon(x)$ is the sign of x and $\theta(x)$ is the step function. In the third equality of (3.13), we used the formula $\int_0^\infty \frac{dt}{t} \sin at = \frac{\pi}{2} \epsilon(a)$. This expression (3.13) is the same as eq.(8.6) in [32], which was obtained by applying the saddle point method directly to the integral (3.6). As is obvious from the derivation of (3.13), this approximation does not capture an essential part of reflection, contained in the t-dependence of $G(t)$. We shall see shortly that $G(t)$ is approximately constant, but only over a finite interval $t < t_0 \sim O(\omega^{-1/2})$. It turns out that the integral (3.12) has most of its support in the region $t > t_0$ when $N \gg \sqrt{\omega}$; a more detailed analysis is thus required.

4. Saddle point approximation of reflection

From the expression (3.12), one can see that the amplitude vanishes linearly as $\omega_i \to 0$, which is an example of the low energy theorem for soft tachyon scattering [32]. Therefore, we expect that the dominant contribution is localized in the middle of the phase space away from the boundary $\{\omega_i = 0\}$, and when N and ω are sufficiently large the amplitude $\mathcal{A}_N$ is sharply peaked around the mean value $\langle \omega_i \rangle = \frac{\omega}{N}$. This in turn implies that the saddle points of the t-integral are located at $\sin \langle \omega_i \rangle t = \pm 1$, i.e. $t = t_l = \frac{\pi(2l-1)}{2\langle \omega_i \rangle}$ $(l = 1, 2, \ldots)$. We checked numerically that the peak around the higher saddle points $t_{l>1}$ easily disappears when ω_i's fluctuate around $\frac{\omega}{N}$, and thus we conclude that the first saddle point $t_1 = \frac{\pi N}{2\omega}$ gives the dominant contribution to the t-integral (3.12).

Numerical evaluation of $G(t)$ shows that it is nearly constant for $t < t_0 \sim O(\omega^{-1/2})$, and exponentially decaying for $t > t_0$ (see figure 5 and below). This in turn leads to a change in the characteristic behavior of the scattering amplitude for $N \gg \sqrt{\omega}$ and $N \ll \sqrt{\omega}$; we therefore analyze these two regimes separately.

4.1 $N \gg \sqrt{\omega}$ case

In the case $N \gg \sqrt{\omega}$, the saddle point $t_1 \sim \frac{N}{\omega}$ is much larger than the characteristic time scale $t_0 \sim \omega^{-\frac{1}{2}}$ of $G(t)$. Therefore, the exponentially decaying behavior of $G(t)$ for $t \gg t_0$ is relavant for the evaluation of the t-integral (3.12). This behavior of $G(t)$ can be reproduced by closing the contour of the ν integral (3.11) in the upper or lower half of the complex ν-plane (depending on the sign of t), and picking up the contribution from the poles of the Γ-functions in (3.5). In this way $G(t)$ is estimated as

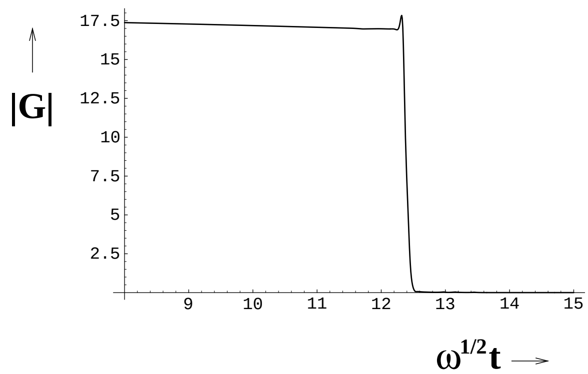

Figure 5: Numerical evaluation of $|G(t)|$ for $\omega = 400$, $q = 0$. The curve for $\omega = 800$, $q = 0$ is virtually identical apart from a rescaling of the vertical scale by $\sqrt{2}$.

$$\begin{aligned} G(t) &\sim 2\sum_{n=0}^{\infty} e^{-(1+q+i\omega+2n)|t|} \frac{(-1)^n}{n!} \frac{\Gamma(1+q+i\omega+n)}{\Gamma(1+q+n)\Gamma(-i\omega-n)} \\ &= \frac{2\sinh \pi\omega}{\pi i} \sum_{n=0}^{\infty} e^{-(1+q+i\omega+2n)|t|} \frac{1}{n!} \frac{\Gamma(1+q+i\omega+n)\Gamma(1+i\omega+n)}{\Gamma(1+q+n)} \\ &= \frac{2\sinh \pi\omega}{\pi i} e^{-(1+q+i\omega)|t|} \frac{\Gamma(1+q+i\omega)\Gamma(1+i\omega)}{\Gamma(1+q)} {}_2F_1(1+q+i\omega, 1+i\omega, 1+q; e^{-2|t|})\,. \end{aligned} \tag{4.1}$$

We should emphasize that this computation of closing the contour does not give an exact answer of $G(t)$ since the integrand of $G(t)$ does not decay as $\nu \to \pm\infty$.

The $n = 0$ term in the summation (4.1) gives

$$G(t) \sim \frac{\Gamma(1+q+i\omega)}{\Gamma(1+q)\Gamma(-i\omega)} e^{-(1+q+i\omega)t}\,. \tag{4.2}$$

This term comes from poles in (3.11) at $\omega \pm \nu = i(1+q)$, where almost all the energy of the initial tachyon is carried by either the particle or by the hole. Including the higher order terms in the sum does not substantially alter the results.

Now the t-integral in (3.12) can be evaluated by expanding the integrand around the saddle point at $t = \frac{\pi N}{2\omega}$ and $\omega_i = \frac{\omega}{N}$

$$t = \frac{\pi N}{2\omega} + u\,, \qquad \omega_i = \frac{\omega}{N} + \epsilon_i \qquad \left(\sum_{i=1}^{N} \epsilon_i = 0,\ |u|, |\epsilon_i| \ll 1\right). \tag{4.3}$$

By performing the gaussian integral over the fluctuation u, $\mathcal{A}_N$ is found to be

$$\begin{aligned} \mathcal{A}_N &\sim \frac{\Gamma(1+q+i\omega)}{\Gamma(1+q)\Gamma(-i\omega)} 2^N \int \frac{du}{\frac{\pi N}{2\omega}} e^{-(1+q+i\omega)(\frac{\pi N}{2\omega}+u)} \exp\left[-\frac{\omega^2}{2N}u^2 - \frac{1}{2}\left(\frac{\pi N}{2\omega}\right)^2 \sum_i \epsilon_i^2\right] \\ &\sim \frac{\Gamma(1+q+i\omega)}{\Gamma(1+q)\Gamma(-i\omega)} 2^N e^{-(1+q+i\omega)\frac{\pi N}{2\omega}} \frac{e^{-N/2}}{\sqrt{N}} \exp\left[-\frac{1}{2}\left(\frac{\pi N}{2\omega}\right)^2 \sum_i \epsilon_i^2\right]. \end{aligned} \tag{4.4}$$

Here we assumed $q \ll \omega$.

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4.2 $N \ll \sqrt{\omega}$ case

In the opposite case $1 \ll N \ll \sqrt{\omega}$, the relevant behavior of $G(t)$ is the plateau region in $t \ll t_0$ (see figure 5). This behavior of $G(t)$ can be obtained by expanding the ν-integral (3.11) around $\nu = 0$. Physically, the point $\nu = 0$ corresponds to the situation where both particle and hole carry the same amount of energy $\frac{\omega}{2}$. When ν is small, the integrand in (3.11) is well approximated by a Taylor expansion in ν

$$\begin{aligned} G(t) &\sim \int \frac{d\nu}{2\pi} F\left(\frac{\omega}{2}\right)^2 e^{it\nu + i\frac{\omega}{\omega^2+q^2}\nu^2} \\ &\sim \frac{1}{2}\sqrt{\frac{i(\omega^2+q^2)}{\pi\omega}} F\left(\frac{\omega}{2}\right)^2 e^{-i\frac{\omega^2+q^2}{4\omega}t^2} \end{aligned} \tag{4.5}$$

with the saddle point

$$\nu_* = -\frac{\omega^2+q^2}{2\omega} t\,. \tag{4.6}$$

This approximation is self-consistent if ν_* is smaller than the width of gaussian in (4.5),

$$|\nu_*| \lesssim \sqrt{\frac{\omega^2+q^2}{\omega}} \quad \Rightarrow \quad |t| \lesssim \sqrt{\frac{\omega}{q^2+\omega^2}}\,. \tag{4.7}$$

Outside of this range, $G(t)$ is highly suppressed due to the oscillatory factor $e^{it\nu}$. Therefore, $G(t)$ is approximated as

$$G(t) \sim \frac{1}{2}\sqrt{\frac{i(\omega^2+q^2)}{\pi\omega}} F\left(\frac{\omega}{2}\right)^2 e^{-i\frac{\omega^2+q^2}{4\omega}t^2} \theta(t_0 - |t|)\,, \tag{4.8}$$

where

$$t_0 = \gamma\sqrt{\frac{\omega}{q^2+\omega^2}}\,. \tag{4.9}$$

γ is a numerical coefficient independent of q and ω. Note that the time difference t is very small $t < \omega^{-\frac{1}{2}}$ in the high energy limit. Putting it all together, in this regime the high energy $1 \to N$ amplitude is given by

$$\mathcal{A}_N = \frac{1}{2} i^{N-1} \sqrt{\frac{i(\omega^2+q^2)}{\pi\omega}} F\left(\frac{\omega}{2}\right)^2 \int_0^{t_0} \frac{dt}{t} e^{-i\frac{\omega^2+q^2}{4\omega}t^2} \prod_{j=1}^{N} 2\sin\omega_j t\,. \tag{4.10}$$

This approximation is consistent when $\frac{N}{\omega} \ll \frac{1}{\sqrt{\omega}}$; beyond this, the Taylor expansion (4.5) breaks down and it is better to use (4.1). Assuming $q \ll \omega$, $\mathcal{A}_N$ becomes (up to a phase factor)

$$\begin{aligned} \mathcal{A}_N &\sim \sqrt{\omega} \int_0^{t_0} \frac{dt}{t} e^{-i\frac{\omega}{4}t^2} \prod_{j=1}^{N} 2\sin\omega_j t \\ &\sim 2^N \sqrt{\frac{\omega}{N}} e^{-i\frac{\omega}{4}\left(\frac{\pi N}{2\omega}\right)^2} \exp\left[-\frac{1}{2}\left(\frac{\pi N}{2\omega}\right)^2 \sum_i \epsilon_i^2\right]. \end{aligned} \tag{4.11}$$

Here we neglected the u-dependence coming from the factor $e^{-i\frac{\omega}{4}t^2}$, since it is subleading when $\frac{N}{\omega} \ll 1$.

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5. The number distribution at high energy

One of the physically interesting quantities we can calculate is the number distribution $\mathcal{P}_N$ of final state tachyons in the $1 \to N$ scattering, which is defined by

$$\mathcal{P}_N = \frac{1}{N!}\frac{1}{\omega}\int_0^\infty \prod_{i=1}^N \frac{d\omega_i}{\omega_i}\delta\left(\sum_i \omega_i - \omega\right)|\mathcal{A}_N(\omega|\omega_i)|^2 . \tag{5.1}$$

$\mathcal{P}_N$ for the low energy and weak coupling scattering has been studied in [32] and it was found that $\mathcal{P}_N$ is almost a Poisson distribution. We are interested in the high energy behavior of $\mathcal{P}_N$.

If we substitute the naive step function approximation of $\mathcal{A}_N$ (3.13) into the definition of $\mathcal{P}_N$ (5.1), it turns out that the resulting ω_i-integral can be performed exactly:

$$\mathcal{P}_{2k-1} \sim \frac{(2\pi)^{2k-1}(2^{2k}-1)|B_{2k}|}{(2k)!\,\omega} . \tag{5.2}$$

Here we used the approximation $|\pi G(0)| \sim \sqrt{\frac{\pi\omega}{2}}$ as in [32].[6] Note that (5.2) is a monotonically increasing function of $N = 2k-1$. It is argued [32] that the approximation (3.13) breaks down near the boundary of step functions. However, one can argue that the contribution from the dangerous boundary region is negligible for small N, so we expect (5.2) to be a good approximation for sufficiently small N.

When N becomes large, (5.2) cannot be trusted anymore and we should use the approximation (4.4) or (4.11), depending on the regime $N \gg \sqrt{\omega}$ or $1 \ll N \ll \sqrt{\omega}$. In both cases, the ω_i-integral can be approximated by the gaussian integral around the saddle point $\langle\omega_i\rangle = \frac{\omega}{N}$

$$\frac{1}{\omega}\int\prod_i \frac{d\epsilon_i}{\frac{\omega}{N}}\delta\Big(\sum\epsilon_i\Big)\exp\left[-\left(\frac{\pi N}{2\omega}\right)^2\sum_i\epsilon_i^2\right] = \left(\frac{2}{\sqrt{\pi}}\right)^{N-1}\frac{N}{\omega^2} . \tag{5.3}$$

Therefore, in the regime $N \gg \sqrt{\omega}$, $\mathcal{P}_N$ is given by

$$\mathcal{P}_N \sim \frac{1}{N!}\frac{\omega^{2q}}{(q!)^2}\left(\frac{8}{e\sqrt{\pi}}\right)^N e^{-(1+q)\frac{\pi N}{\omega}} \tag{5.4}$$

and similarly in the regime $N \ll \sqrt{\omega}$

$$\mathcal{P}_N \sim \frac{1}{N!}\frac{1}{\omega}\left(\frac{8}{\sqrt{\pi}}\right)^N . \tag{5.5}$$

The discrepancy in these two formulae for $N \sim \sqrt{\omega}$ is subleading in N, and can be attributed to the different approximations made in the two regimes.

[6] Curiously, there is a nice answer for the generating function of this result:

$$\sum_{k=1}^\infty \mathcal{P}_k x^k = (2\omega)^{-1}\tan(\pi x) .$$

We also find it remarkable to discover yet another appearance of the seemingly ubiquitous Bernoulli numbers.

6. Discussion

The main observation is that the probability to have N outgoing particles is a monotonically and steeply decreasing function of N, at least when N is sufficiently large. On the other hand, (5.2) suggests the probability grows with N for N small; indeed, in [32] the probability to have more than one particle in the final state at high energy was estimated by a unitarity argument[7] to be (noting that $\mathcal{P}_1 \sim \frac{\pi}{2\omega}$ according to (5.2))

$$\mathcal{P}_{N>1} = 1 - \mathcal{P}_1 \sim 1 - \frac{\pi}{2\omega} \,. \tag{6.1}$$

Taken together, our results suggest that the distribution is peaked for some reasonably small number of outgoing particles, larger than one. Note that equation (5.2) gives $\mathcal{P}_N \sim O(1)$ for $N \sim \log \omega$; we therefore expect $\mathcal{P}_N$ to be peaked in this regime.

There is no characteristic feature for $N \sim \omega$ that might indicate the presence of a black hole or similar object as an intermediate state. Instead, the configuration that dominates high energy scattering is the motion of a single particle-hole pair bouncing off the potential. For $N \gg \sqrt{\omega}$, the energy is carried predominantly by either the particle *or* the hole; for $N \ll \sqrt{\omega}$, it is shared roughly equally by each.

A qualitative explanation for the rapid decrease of $\mathcal{P}_N$ with N comes from the fact that the N tachyon state is built from fermion bilinears $\psi^\dagger\psi$. The initial state consists of one such bilinear, which then picks up a phase from reflection off the potential (1.4). The final state is a product of N such bilinears, and the overlap with the evolved initial state picks out the terms in $\psi^\dagger\psi \cdots \psi^\dagger\psi$ where all but two of the fermion operators act on one another. This is a relatively small fraction of the overall possibilities, since this operator can create up to N particle-hole pairs when acting on the vacuum. The initial state selects only the part with one particle-hole pair.

6.1 Comments on conservation laws

The free fermionic character of the matrix model leads to an infinite set of conserved quantities; the time-independent ones are

$$\mathcal{Q}_\ell = \int d\varepsilon \; \varepsilon^\ell \, b_\varepsilon^\dagger b_\varepsilon \tag{6.2}$$

where $b, b^\dagger$ are fermion creation and annihilation operators. Typically, black holes have no hair, suggesting that their conserved charges are $\mathcal{Q}_1 = \omega$, $\mathcal{Q}_\ell = 0$.[8] States with these properties in the free fermion Hilbert space consist of a macroscopic number of very soft tachyon excitations — more or less a coherent state of soft tachyons.[9] On the other hand,

[7] Unitarity also tells us that the probability to send in a single initial tachyon and then find N tachyons in the final state, is the same as that for the time reversal prosess of $N \to 1$ scattering. Intuitively, it seems unlikely to have a single tachyon state as the final state of N tachyon scattering with large-N, unless we fine-tune the initial state.

[8] Results to this effect were presented by A. Sen at Strings2004 [37]. See also [38].

[9] This feature is in accordance with higher dimensional examples, where to form a macroscopic black hole requires the excitation of a macroscopic number of branes.

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our one-particle initial state has $\mathcal{Q}_\ell \sim \omega^\ell$. In order to form a black hole as an intermediate state, an initial state consisting of a small number of high energy tachyons would have to shed these higher conserved charges, e.g. by radiating a shell of outgoing tachyons carrying them away, leaving behind an intermediate state with the characteristics of the black hole.

The phase space available to soft particle-hole pairs is not large, and the only objects with the charges $\mathcal{Q}_\ell \sim \omega\delta_{\ell,0}$ are collections of soft fermion excitations of energies $\{\omega_i\}$, $i = 1, \ldots, N$, relative to the filled Fermi sea;[10] these yield the conserved charges

$$\mathcal{Q}_1 = \sum_{i=1}^{N} \pm\omega_i = \omega\,; \qquad \mathcal{Q}_\ell = \sum_{i=1}^{N} \pm\omega_i^\ell \sim o(N^{1-\ell})\,, \qquad \ell > 1 \tag{6.3}$$

with the sign $+$ for particles and $-$ for holes. Since there is no exponentially large phase space of black hole states, the evolution is not drawn towards the formation of an intermediate state with the conserved charges associated to the black hole — there are not sufficiently many states to make visiting such a configuration likely.

Now, to create an intermediate state corresponding to a finite deformation of the Fermi sea, such as a blob of eigenvalues separated away from the sea, one must consider a more general $N' \to N$ scattering with $N', N \gg 1$, instead of the $1 \to N$ scattering studied in this paper. This is because the number of particle-hole pairs appearing in the $N' \to N$ scattering is bounded from above by $\min\{N', N\}$. In particular, only a single particle-hole pair contributes in the $1 \to N$ scattering as we saw above, and therefore the initial state we considered has rather small overlap with the states carrying the charges expected for a black hole state. But if it happens that most high energy states do not scatter into black holes, then this casts doubt on the existence of any object that one should call a black hole.

The intuition gained from our study of $1 \to N$ scattering shows why this soft-excitation state will not have the properties of a black hole. The Hawking radiation of a black hole consists predominantly of quanta with energy of order the limiting temperature $T_{\rm H} = \frac{1}{2\pi}\sqrt{\frac{2}{\alpha'}}$. Such an outgoing state would have to be constructed from an intermediate state consisting of $N \gg \omega$ quanta, in order that the charges (6.3) approximate those of a black hole, $\mathcal{Q}_\ell = \omega\delta_{\ell,0}$; again, the probability that $N/\omega \equiv n \gg 1$ soft particle-hole excitations will make a Hawking quantum behaves like the overlap of a state created by $(\psi^\dagger\psi)^n$ and one created by $\psi^\dagger\psi$, which we expect to be a rapidly decreasing function of n.

6.2 Whither the 2D black hole?

The evidence seems to suggest that there is no such object as the 2D black hole in string theory. The question then is why we were misled into thinking so by the seemingly exact coset construction of a worldsheet CFT describing strings propagating in a black hole background.[11] Consider the euclidean continuation of (1.3), in which the putative euclidean black hole geometry is a capped semi-infinite cylinder often referred to as the "cigar". This geometry is strongly curved in the vicinity of the tip of the cigar for the regime of interest,

[10] So that $\omega_i < 0$ for holes and $\omega_i > 0$ for particles.

[11] We thank David Kutasov for the following argument; see [39, 40].

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$k \sim 2$. In fact, there is a conjectured strong-weak coupling duality [41, 42, 39] between the 2D black hole and Sine-Liouville theory, i.e. the tachyon background

$$T = \lambda\, e^{\frac{1}{Q}\phi} \cos\theta \tag{6.4}$$

describing a condensate of vortices, where θ is the (axial) target euclidean time coordinate, and $Q = (k-2)^{-1/2}$. The euclidean cigar geometry is a weakly coupled sigma model for $Q \to 0$, while the Sine-Liouville background is weakly coupled for $Q \to \infty$. The regime of interest for 2D black holes is the strongly coupled one, where the Sine-Liouville description is more appropriate, and one cannot use the cigar geometry to extract reliable predictions. This observation is consistent with the fact that the Hagedorn thermodyamics of high mass black holes is not to be found in the matrix model. The Sine-Liouville background appears to describe instead a classical tachyon background of the 2D string, consistent with the picture deduced from the consideration of conserved charges in the previous subsection.

Perhaps the proper lesson to be drawn from all the known facts is that high energy scattering is dominated by brane physics. In higher dimensions, brane physics is black hole physics — the degrees of freedom on brane intersections lead to a large degeneracy of black hole intermediate states, which dominate the S-matrix. In two dimensions, the high energy scattering is still dominated by branes — the D0-branes that are the eigenvalue fermions of the matrix model — but the kinematics is so restrictive that they do not have a black hole density of states. Instead of forming a black hole, a highly energetic tachyon in a sense becomes a D0-brane that bounces high off the background potential of figure 2, and then fragments into a small number of very energetic tachyons in the final state.

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A. The $2 \to N$ amplitude

The $2 \to N$ amplitude is also written in terms of the function $G(t)$ in (3.11). When $\mu = 0$ and $N =$ even, the $2 \to N$ amplitude is given by

$$\begin{aligned}
&\mathcal{A}_{2\to N}\left(\frac{\omega}{2},\frac{\omega}{2}\middle|\,\omega_i\right) = \\
&= \frac{1}{\pi}\int_0^\infty \frac{dt}{t}\int_0^\infty \frac{ds}{s}\Bigg\{ G_\omega(t)\left[\prod_{l=1}^{N} 2i\sin\omega_l(s+t) - \prod_{l=1}^{N} 2i\sin\omega_l(s-t)\right] - \\
&\qquad - \sum_{A\cup B=\{1,\dots N\}} G_{\omega_A} * G_{\omega_B}(t)\left[\prod_{l\in A} 2i\sin\omega_l(s+t) - \prod_{l\in A} 2i\sin\omega_l(s-t)\right]\times \\
&\qquad\qquad \times \prod_{j\in B} 2i^{-1}\sin\omega_j s\Bigg\}\,.
\end{aligned} \tag{A.1}$$

Here $f * g(t)$ is the convolution

$$f * g(t) = \int_{-\infty}^{\infty} du f(u) g(t-u) \,. \tag{A.2}$$

Qualitatively, this amplitude will have a similar structure to the $1 \to N$ amplitude. Both G and $G * G$ only have appreciable support for $t < O(\omega^{-1/2})$, and the sine factors are qualitatively similar to the $1 \to N$ result. Thus we expect qualitatively similar behavior, namely a monotonically decreasing probability for finding N particles in the final state as a function of N.

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